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A NOTE ON CONSUMER SURPLUS WITH QUALITY VARIATION

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WP# 1938-87 Revised July 1986

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In this paper we extend the consumer surplus[1] concept, to compare equilibria characterized by both price and quality changes. There are numerous situations where it is possible to provide a high quality-high price service or a lower price-low quality service; the problem that arises is evaluating the welfare loss or gain in moving from one equilibrium to another. Examples of such situations are, for instance, those faced by public utilities (determining the quality of service, in this case the probability of power failure); the decision to set quality standards by telephone companies (the probability of a call going through); the service provided by a computer center (the probability of achieving a certain turnaround time), highway design (the probability of getting from point A to point B in a certain time) and the water board (the purity of the water supplied).

As an illustration, consider the decision to be made by a telephone company in setting the quality of service, in this case measured by the probability of a call getting through. Clearly, the management of the company has the option of providing a high probability of a call getting through at a higher price or charging a lower price concomitant with a lower probability of a call getting through. The problem facing the company is the determination of an optimal quality or at least of finding a way to compare two different price-quality equilibria. In the U.S., the fact that a call almost always goes through just goes to show that the quality of service is high. This is hardly the norm for the rest of the world [consider for example direct dialing between Delhi and Bombay]. Casual observation shows that the probability of a direct dialled call from New York to New Delhi getting through in the first attempt is less than 0.3, whereas for a similar call to the United Kingdom the probability exceeds 0.9. Clearly there are conscious decisions on the part of the phone company. If, as a regulated monopoly, it is acting in the best interest of the community, it should have some means to determine an alternative price-quality pair than that with which it currently functions.

As is shown (in Section 3) a substantially greater amount of information is required to evaluate changes in equilibria accompanying simultaneous price and quality changes than in the case when only price varies. In the latter case it is sufficient to know the demand curve between the initial and final prices. However, when both price and quality change, we require knowledge of the demand curves from a price at which demand is zero to the price at equilibrium. Although this poses no serious problem at a theoretical level, this extensive information is typically not available to someone who wants to evaluate equilibria characterized by changes in both price and quality.

Section 4 of this paper discusses how local changes in price and quality can be compared without knowledge of the demand curve. The technique involves the observation of an equilibrium and uses a revealed preference approach. Since it is based on readily available, observable information, it becomes a powerful operational procedure.

2. The Economy.

The economist's attitude towards quality is typified by the following remark.

> "Two good stuffs of the same kind may differ in flavour or nutritive content... However, the concept of a commodity can be adapted to this diversity amongst products of the same kind. Two different qualities of the same product or service may in fact be represented by two different commodities". (Malinvaud [1972]).

In other words, Jaffa oranges and Florida oranges are two different commodities as are the services of two computer centers, identical in every respect except that the users' jobs have different expected waiting times in line before processing in the two centers. So, strictly speaking, when we compare equilibria characterized by both a price and a quality change we are dealing with two different goods. When the quality of a commodity or a service changes it is trans-

formed into a different commodity or service. Hence, unlike the case considered in the traditional consumer surplus literature, when both the price and quality change, the initial and final equilibria are on two different demand curves. This is the point of divergence between the two cases (i.e., the case when only price changes and when both price and quality change).

Assumptions and notation:

I) There are N different kinds of commodities or services (we will use the terms interchangeably) in the economy. A commodity point is represented by $d = (d_1, d_2 \ldots . d_N)$.

II) With each kind of commodity d_j is associated a quality q_j, where q_j is a rating on a quality scale. For the type of problems under consideration, quality can be measured meaningfully on a cardinal scale. In the examples cited earlier, quality was related to reliability (probability of failure) and efficiency (probability of performing a task within a specified time). In both cases the cardinality interpretation is justified and permits quality variations to be quantified in a meaningful way (see Section 4).

III) There are I different individuals; with each individual i there is associated an exogenously specified income m_i, a utility function u_i and an indirect utility function $\phi_i(p,q,m_i)$, where ϕ_i is the maximum utility available given the price vector $p = (p_1, p_2, \ldots p_N)$, the quality vector q and income m_i.

$$\text{By definition}^2 \quad \phi_i(p,q,m_i) = \max \, u_i\{d_1(q_1), d_2(q_2), \ldots, d_N(q_N)\}$$

$$\text{subject to} \quad \sum_{j=1}^{N} p_j(q_j) d_j(q_j) \leq m_i .$$

When we are dealing with a single individual the index i will be dropped.

IV) The Marshallian demand of individual i for commodity j is represented by $d_{ij}(p,q,m_i)$. In the interest of simplicity we will suppress variables that are constant, so that whenever we consider prices and qualities of goods other than j fixed, we will write $d_{ij}(p,q,m_i)$ as $d_{ij}\ (p_j, q_j, m_i)$ or when there is no formed into a different commodity or service [illegible]

in the conditioned consumer surplus literature [illegible]

change, the [illegible] total expenditure [illegible]

This is the [illegible]

price and [illegible] quantity [illegible]

Assumptions and [illegible]

i) There are [illegible] commodities [illegible]

interchangeably [illegible] commodity quality [illegible]

$d = (d_1, d_2, \ldots)$ [illegible]

ii) [illegible]

ratio [illegible]

can be [illegible]

quality [illegible]

utility [illegible]

as [illegible]

in a maximum [illegible]

iii) The [illegible]

an [illegible]

function [illegible]

vector [illegible]

$v(p,q)$ [illegible]

subject to $\sum_{i=1}^{n}$ [illegible]

When we are dealing with a single [illegible]

vi) The [illegible]

$x_{ij}(p,q,m)$ [illegible]

are constant, so that [illegible]

that [illegible], we will write $x_i(p,q,m)$ [illegible] ambiguity, simply as d_{ij}. The demand curve d_{ij} is assumed to be differentiable with respect to p_j and q_j.

V) It is assumed that there exists a price p_j^∞ such that $d_{ij}(p_j^\infty, q, m_i) = 0$ for all i $(i = 1,2,\ldots I)$ i.e. for each kind of commodity j and all individuals i there exists a price (p_j^∞) independent of the quality such that the demand for that commodity is zero.

3. Consumer Surplus with Quality Variation: Variation in the quality and quantity of one commodity (j).

Consider the indirect utility function for individual i

(1) $\phi_i(p,q,m_i)$.

The demand d_{ij} for commodity j can be expressed as

$$d_{ij}(p_j,q_j,m_i) = -\frac{\partial\phi_i}{\partial p_j} \Big/ \frac{\partial\phi_i}{\partial m_i}. \qquad (2) \quad \text{(Roy's identity)}$$

Suppose commodity j can be supplied at price $p_j = p_j^1$ with quality $q_j = q_j^1$. We begin by estimating the benefit to the consumer of being able to buy commodity j at price p_j^1 and quality q_j^1 rather than at price p_j^∞. (i.e., not being able to buy the commodity j at all). Figure 1 shows the demand curve for commodity j with quality q_j^1 (assuming all other prices and qualities are fixed).

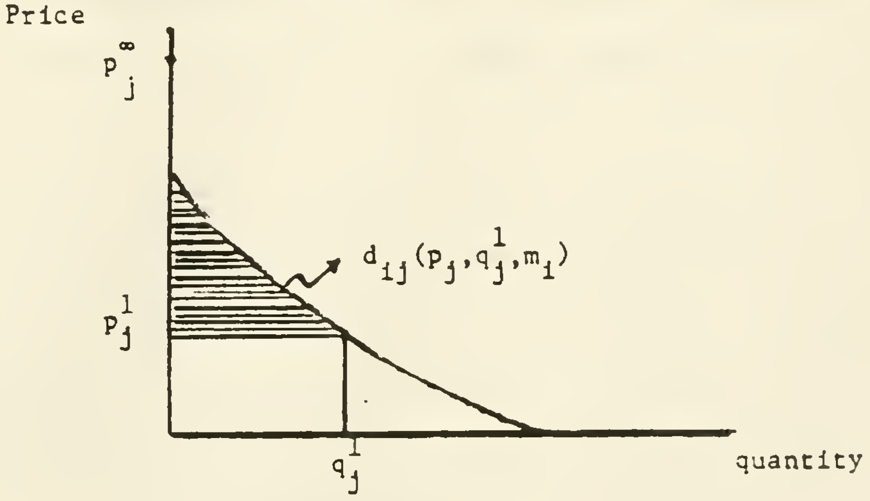

Figure 1

Consider the shaded area, which can be expressed by

$$C_i = \int_{p_j^1}^{p_j^\infty} d_{ij}(p_j, q_j^1, m_i)dp_j$$

or

$$(3) \qquad C_i = \int_{p_j^1}^{p_j^\infty} \left(- \frac{\partial\phi_i}{\partial p_j} \Big/ \frac{\partial\phi_i}{\partial m_i}\right)dp_j.$$

If we assume that the marginal utility (MU) of income $\partial\phi_i/\partial m_i$ is independent of the price commodity j [3] and let $\frac{\partial\phi_i}{\partial m_i} \equiv \lambda_i$, then

$$(4) \qquad C_i = \frac{1}{\lambda_i}[\phi_i(p_j^1, q_j^1, m_i) - \phi_i(p_j^\infty, q_j^1, m_i)].$$

Since the individual does not consume any good when the price is p_j^∞

$$(5) \qquad \phi_i(p_j^\infty, \cdot, \cdot) = \gamma_i \quad \text{(a constant).}$$

Therefore we can rewrite (4) as

$$(6) \qquad C_i = \phi_i(p_j^1, q_j^1, m_i)/\lambda_i - \gamma_i/\lambda_i.$$

C_i can be interpreted as the increase in utility achieved by reducing the price of commodity j, converted into money units by division by the marginal utility of income λ_i.

If instead of using the ordinary (Marshallian) demand function we had used the Hicksian demand function (compensated demand function) for commodity j then C_i would have given an exact estimate of the income it would be necessary to take from individuals so that they remained at a constant utility level following a price decrease from p_j^∞ to p_j^1. (i.e. it would give the compensating variation (Hicks [1956]). In this case, the assumption that the marginal utility of income λ is independent of p_j would not be required.

It is well known that if the income effect is small[4] the Marshallian demand function approximates the Hicksian demand function. (If the income effect is zero the two are identical.[5]) Hence if the income effect of commodity j is small or if the marginal utility of income is independent of the price of commodity j, then the area C_i has a ready welfare interpretation. In the analysis that follows one of the conditions above is assumed to hold.

Next we discuss how two equilibria characterized by different prices and qualities can be compared.

Assume that commodity of type j can be provided at price p_j^1 and quality q_j^1, or at price p_j^2 and quality q_j^2 and that we wish to compare the two alternatives.

Let

$$C_i^1 = \int_{p_j^1}^{p_j^\infty} \left(- \frac{\partial \phi_i(p_j, q_j^1, m_i)}{\partial p_j} \Big/ \lambda_i \right) dp_j$$

(7) or $C_i^1 = \phi_i(p_j^1, q_j^1, m_i)/\lambda_i - \gamma_i/\lambda_i$ (using equation 5)

and $$C_i^2 = \int_{p_j^2}^{p_j^\infty} \left(- \frac{\partial \phi_i(p_j, q_j^2, m_i)}{\partial p_j} \Big/ \lambda_i \right) dp_j$$

(8) or $C_i^2 = \phi_i(p_j^2, q_j^2, m_i)/\lambda_i - \gamma_i/\lambda_i$.

In view of the preceding discussion C_i^1 & C_i^2 represents the increase in utility (converted into monetary units) achieved by reducing the price of commodity j from p_j^∞ to the price-quality pairs (p_j^1, q_j^1) and (p_j^2, q_j^2) respectively.

Hence for individual i the pair

(9) $(p_j^1\ q_j^1) \succ (p_j^2\ q_j^2)$ if and only if $C_i^1 > C_i^2$.

Figure 2 illustrates the situation graphically.

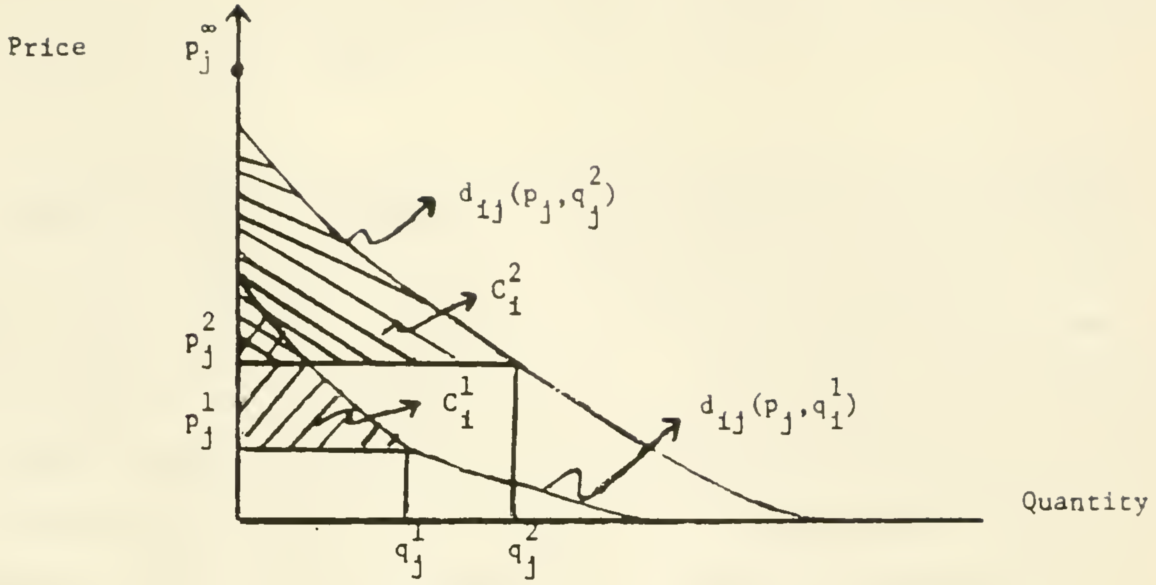

Figure 2

Since, as mentioned earlier, at a price p_j^{∞} the individual does not consume any amount of the commodity j (irrespective of quality), we can use equation 5 to compare all equilibria associated with different price-quality pairs of a particular good j with reference to p_j^{∞}. Note that in the traditional consumer surplus literature such a reference point is not required, since in that case we move along a demand curve, and to compare two equilibria only the portion of the curve between the two prices is required. Another point to note is that while in the case when only price varies the situation with a lower price will always be preferred by an individual, this is not necessarily true when both price and quality are allowed to vary.

When there are more than one individuals

$$(10) \qquad (p_j^1,q_j^1) \succ (p_j^2 \;\; q_j^2)^{6} \text{ if and only if } \sum_{i=1}^{I} C_i^1 \geq \sum_{i=1}^{I} C_i^1$$

assuming we weight each individual equally. Equation 10 will hold since we can always devise a scheme such that those who gain can adequately compensate the losers.

[illegible]

[illegible] and [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] good [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] cuts [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] case we have a [illegible] demand curve, and [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] the two [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] while [illegible] [illegible] only [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] always be [illegible] [illegible] [illegible], this [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] very.

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

(10) [illegible] and [illegible]

assuming we [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] always [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

We can easily extend the analysis to the case where n pairs of prices and qualities vary (see Mehra (1985)).

4. Evaluating benefits associated with local changes in price and quality

The concept developed in section 3 requires that the upper half of the relevant demand curves be known (see figure 2) before any comparisons between different price and quality pairs can be made. In this section we will show how local changes in price and quality around an OBSERVED equilibria can be evaluated.

Consider a representative individual at an initial equilibrium where he is consuming $d_j(p_j^o, q_j^o)$ units of commodity j at a price p_j^o and quality q_j^o. In this section, in the interest of clarity, we will suppress the index i for the individual and all other arguments of d_j $(\cdot)$ that are fixed. We rewrite equation 3 as follows

$$B(p_j,q_j) = \int_{p_j}^{p_j^\infty} d_j(p_j,q_j)dp_j.$$

We can expand $B(p_j,q_j)$ around the initial equilibrium (p_j^o, q_j^o) by a Taylor expansion.

$$(11) \quad B(p_j,q_j) \approx B(p_j^o,q_j^o) - d_j(p_j^o, q_j^o)(p_j - p_j^o) + \left[\int_{p_j^o}^{p_j^\infty} \frac{\partial d_j(p_j^o, q_j^o)}{\partial q_j} dp_j \right] (q_j - q_j^o)$$

to a first order approximation.

Let $(p_j^1\ q_j^1)$ be another point within the range of the expansion.

$$\therefore \quad B(p_j^1, q_j^1) - B(p_j^o, q_j^o) \approx - d_j(p_j^o, q_j^o)\ (p_j^1 - p_j^o) + \left[\int_{p_j^o}^{p_j^\infty} \frac{\partial d_j(p_j^o, q_j^o)}{\partial q_j} dp_j\right](q_j^1 - q_j^o).$$

$$\text{Let} \quad J = \int_{p_j^o}^{p_j^\infty} \frac{\partial d_j(p_j^o, q_j^o)}{\partial q_j} dp_j.$$

If $B(p_j^1, q_j^1) \sim B(p_j^o, q_j^o)$ where $\sim$ denotes indifference, then

$$J \approx d_j(p_j^o, q_j^o)[p_j^1 - p_j^o] / [q_j^1 - q_j^o].$$

Once J is evaluated equation (11) is fully known in terms of observables. Note (p_j^1, q_j^1) need not be an equilibrium price-quality pair. Therefore, to evaluate local changes around (p_j^o, q_j^o) we need another price-quality pair (p_j^1, q_j^1) such that the individual is indifferent between the two pairs. Such information could be elicited from individuals using an appropriate questionnaire.

For a particular individual $(p_j, q_j) \succsim (p_j^o, q_j^o)$ if and only if $B(p_j, q_j) - B(p_j^o, q_j^o) \geq 0$.

If, as is typically the case that there is more than one individual, the demand curves and hence J, B(·, ·) and ΔB would be indexed by i (i = 1, 2, ..., I). In this setting J_i would have to be evaluated for each individual and $(p_j, q_j) \succsim (p_j^o, q_j^o)$ if and only if $\sum_{i=1}^{I} [B_i(p_j, q_j) - B_i(p_j^o, q_j^o)] \geq$ assuming each individual is weighted equally.

In practice the utility company, the phone company or the computer center could use a stratified sampling of its customers based upon the characteristics of the population such as the current level of their demands, before distributing the questionnaire. This procedure would be similar to those that are commonly used in marketing research studies. Note that to calculate J_i the relevant decision making authority already has information about $d_{ij}(p_j^o, q_j^o)$, p_j^o, q_j^o. For example, the electric company already knows the current demand by individual i, d_{ij}, the current price p_j^o and the current quality q_j^o. Thus this process enables meaningful and accurate decision making based upon a manageable and readily available body of data.

This analysis can readily be extended to the case of local changes in the quality of n commodities (see Mehra (1985)). Though clearly the most interesting and tractable case is the one discussed above where n = 1.

As shown in Mehra (1985) in the special case when quality does not vary

$$(p, q) \succ (p^o, q^o) \text{ if and only if } \sum_{i=1}^{I} \sum_{j=1}^{n} d_{ij}^o \, (p - p^o) \geq 0.$$

5. Conclusions

This paper extends the consumer surplus concept so that the benefits associated with a given change in price and quality can be evaluated. The difference between the traditional consumer surplus concept and the concept developed here are pointed out. Finally it is shown how local changes in price and quality can be evaluated on the basis of observables without the knowledge of the demand curve. Essentially we use indifference information in the region of the Taylor expansion to estimate how the indirect utility function varies locally. Since we are not concerned with total surplus but only with changes, we could have obtained the same information using the utility foundation to construct the entire demand curves (Eq. 7 and 8 or 11). However, in section 5 we show that the knowledge of the full demand curve is unnecessary and although we require all the information to extrapolate the full demand curve, we never need to write it out.

FOOTNOTES

1. The interested reader is referred to the survey paper by Currie, Murphy and Schmitz (1971), the papers by Burns (1973), Hicks (1956), Mishan (1972), Silberberg (1972), Willig (1976) and Zajac (1976) which succinctly expound the concept and to Spence (1975) for addressing the issue of setting optimal quality under monopolistic competition.

2. The notation $d_j(q_j)$ is used to stress that demand is a function of quality. It should not be interpreted as suggesting that demand for good j is ONLY a function of quality.

3. Note the marginal utility of income need not be independent of the prices of the other N-1 commodities. Samuelson (1942) has shown that the assumption that $\partial\phi/\partial m$ be independent of all N prices leads to absurd results.

4. See Willig (1976) for the accuracy of this approximation. The percentage error from using the area under the Marshallian demand curve (A) rather than the area under the Hicksian demand curve (C) is

$$\frac{C-A}{A} \approx \frac{\eta A}{2m}$$

where η is the income elasticity. This is likely to be small in practice.

5. This follows from Slutsky's equation

$$\frac{\partial d_j(p,q,m)}{\partial p_j} = \frac{\partial h_j(p,q,\phi)}{\partial p_j} - \frac{\partial d_j(p,q,m)}{\partial m} .d_j$$

where $h_j(p,q,\phi) = \dfrac{\partial\theta(p,q,\phi)}{\partial p_j}$; $\dfrac{\partial d_j(p,q,m)}{\partial m} .d_j$ is the income effect

and $\theta(p,q,u^o) = \min \sum_{j=1}^{N} p_j(q_j)d_j(q_j)$

subject to $u\{d(q_1),d(q_2),\ldots d(q_N)\} > u^o$.

6. $>$ denotes weakly preferred.

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